

# **Quantifying Our Ignorance: Stochastic Forecasts of Population and Public Budgets**

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# Quantifying Our Ignorance: Stochastic Forecasts of Population and Public Budgets

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A good deal of demographic research is justified on the grounds that it may lead to improved population forecasts. However, most researchers have never made a forecast, and actual projections are usually done by some form of trend extrapolation with little mention of theory. I believe that most demographers view forecasting as a mechanical exercise having little intellectual content. And indeed much forecasting, including good forecasts and my own forecasts, could be fairly characterized in this way. But nonetheless forecasting is one of the most important tasks demographers perform, and it is important that it be done well and to high professional standards. We need forecasts to anticipate population aging, for example, and as inputs for economic, fiscal, environmental, and social service planning. And we need forecasts simply to be able to visualize our collective future. Some kinds of planning depend on demographic patterns many decades in the future, and because of the long-term demographic consequences of current population age distribution, demographers sometimes make useful predictions of these. However, like most kinds of forecasts, population forecasts often turn out to be quite mistaken. It is also a task of demographic forecasters to provide indications of the kinds of errors they may make and the probabilities of such errors.

Demographers use accounting identities to translate assumptions about the time path of age-specific fertility, mortality, and migration into the future population sizes and age distributions they imply. However, demographic rates are only probabilities at the individual level. If a fertility rate for a 27-year-old women is 0.5 per year, that means there is a 50 percent probability that a particular woman will give birth within a year, and a 50 percent chance that she will not, and similarly for probabilities of death and survival, and migration. This intrinsic uncertainty at the individual level is diminished when we talk about larger groups of women, because it tends to average out, but it never disappears completely. Even if the true rates

were known with certainty *ex ante*, the outcomes could not be predicted with certainty. Early researchers looked here for the source of uncertainty in population forecasts. However, simple calculations showed that this source of uncertainty became vanishingly small in the larger national populations, yet forecasts for large populations were little more successful than for small ones. Then it was recognized that the main source of uncertainty was that the vital rates (that is, the probabilities) themselves change over time, and that we make errors in forecasting these changes (Sykes 1969).

Unfortunately, the analytic tools of pure demography are of little use for the task of forecasting changes in vital rates. The principal analytic technique available is to place current rates in a longer run context through skilled disaggregation—for example, by parity and length of open birth interval for fertility, or by cause of death for mortality. But disaggregations of this kind are useful mainly when the underlying, disaggregated probabilities are in fact unchanging and the changes in outcomes are due to changing structures, that is to changing distributions of the population across the different relevant disaggregated categories of risk. However, there is an underlying weakness with this approach: If the distributions are not changing, then the disaggregation is not helpful. On the other hand, if the distributions are irregular and changing, this must reflect past variations in the disaggregated rates. But if the rates have been changing in the past, then they are likely to change in the future, and we are back to the problem of trying to forecast their changes. Faced with the need to forecast the disaggregated rates, demographers tend to extrapolate their most recently observed levels. Thus for the most part, classic demographic methods are of limited use in a changing world.

For the past 35 years, I have been working on the problem of demographic forecasting under uncertainty. My goal has been partly intellectual, to figure out how to think about the processes at work. But it has also been to develop new forecasting methods that would be more than illustrative and that might be good enough to inform real-world planning.

In this article, I will explain how this work developed over the years, what directions I tried, and why some failed and some succeeded. I will not try to provide a comprehensive overview of this large topic, which I attempted to do in Lee (1999) and Lee and Tuljapurkar (2000). However, I will give some indication of the contributions of others, and the directions they have taken.

## Fundamental issues

As a graduate student, my interest in forecasting stemmed from an attempt to understand how and why populations grew and declined, and how the process was related to economic change, age distribution, and accident. In thinking about these questions, I was troubled by a number of questions:

1) Theories of Malthus, Easterlin, ecologists, and others tell us that population size and age distribution should themselves have an influence

on current vital rates, which implies that population processes should be subject to negative feedback. Yet in practice this is generally ignored in projections. What should we make of this?

2) Some analysts have viewed population change as an independent force, itself explaining economic and social changes. Others have viewed population change as endogenous, responding passively to economic change. How can we reconcile the interplay of random and systematic influences on vital rates and population growth?

3) How can variations over age and time in vital rates such as fertility or mortality be represented parsimoniously yet realistically?

Regardless of how forecasting is actually carried out, it explicitly or implicitly involves deep theoretical and empirical assumptions and judgments. Can we ignore feedback? Do more people in the reproductive ages mean more births or fewer births? Will environmental pressures be increased by population growth and in turn retard that growth? Will mortality rise as population density increases? Does a larger population lead to more rapid technological progress, or to poverty? If, in the end, most population forecasts say nothing about these questions, that outcome reflects an assumption on the part of the forecasters that demographic trends can continue without encountering such feedback, not proof that such economic and environmental interactions with population do not matter. The potential role of such factors is discussed in Lee (1990) and Cohen (1995).

### Modeling variation in demographic rates over age and time

There is a tradition in mathematical demography of modeling variation over age and time, closely related to the demographic tradition of constructing model age schedules for fertility, nuptiality, and mortality. One approach involves fitting nonlinear parametric functions to the observed age schedules and then letting some or all of the parameters vary over time. For example, Gompertz or gamma functions have been used for fertility, and the Heligman–Pollard nine-parameter function has been used to extrapolate mortality rates (McNown and Rogers 1989). Another approach generates new age schedules by transforming an existing one, or a standard; this is sometimes called a relational approach. For example, one simple model assumes an equal additive change to mortality at every age, which is the simplest and most tractable model of all, but which fits actual change only poorly. Another example, this time more realistic but less tractable, assumes that mortality at every age changes by the same proportion. Still more realistic is Brass's logit transform.

The approach I developed was still very simple but, like the logit, both flexible and realistic. It lets each age-specific rate have its own additive or

multiplicative pattern of change, with the relative sizes of these changes fixed across age:

$$m(x,k) = a(x) + k*b(x).$$

Here  $m(x,k)$  can represent either a vital rate at age  $x$  for parameter value  $k$ , or the logarithm of the rate for the multiplicative version. As  $k$  varies, the model generates a family of age schedules of the vital rates  $m$  or  $\exp(m)$ . The model can be fit to a matrix of historical rates varying over age and time, resulting in estimates of the  $a(x)$ ,  $b(x)$  schedules, and a time series of the parameter  $k$ , call it  $k(t)$ . We can hold  $a(x)$  and  $b(x)$  fixed and then focus on analyzing, modeling, and forecasting  $k(t)$  without having to worry about the age-specific details. The model has been used mainly for mortality, but also for fertility and migration.

This model also makes it easy to incorporate stochastic disturbances in a natural way, by treating  $k$  as a random variable. Then  $k$ , for fertility or mortality, can be projected using standard statistical methods, together with its probability distributions. From these, probability distributions for age-specific fertility and mortality can be calculated. Other sources of uncertainty, resulting from imperfect fit of the model given above, or from uncertainty about the parameters of the time series models used to forecast  $k$ , can also be incorporated.

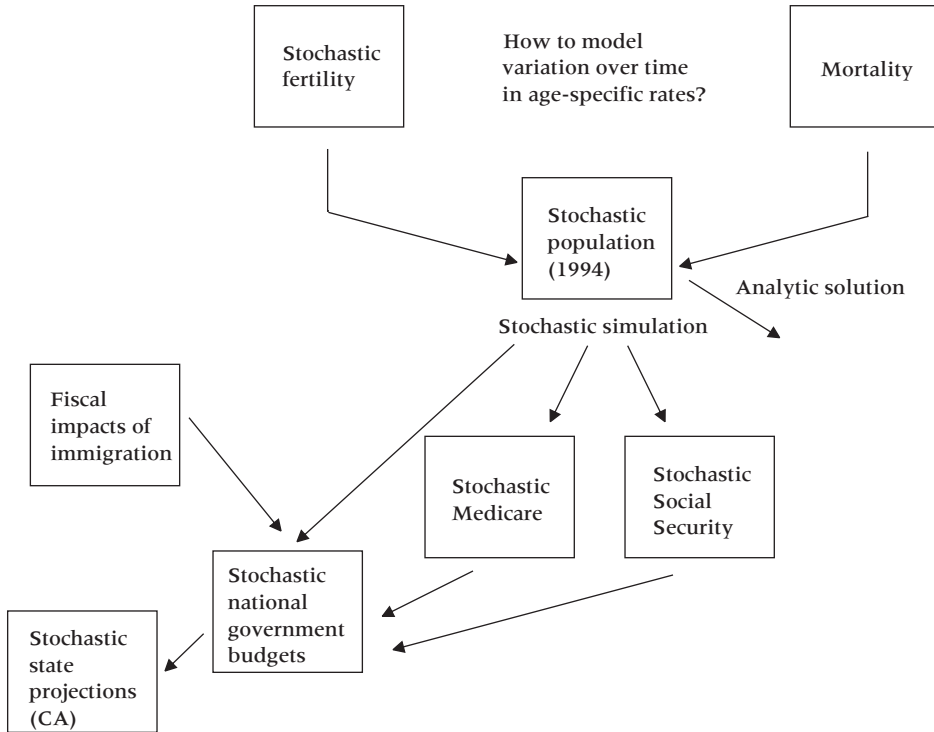
## The framework

Figure 1 gives a very large-scale schematic framework for this multi-decade undertaking, showing fertility and mortality as processes to be modeled, then showing them as inputs to a stochastic population forecast, and finally showing the use of the stochastic population forecast as an input to stochastic fiscal projections of various kinds. The vertical arrangement in Figures 1, 3, and 4 is both temporal, with the earlier work higher up, and logical, with the earlier entries providing inputs for entries lower down in the figures. The remainder of the article takes a close look at each of these legs of the journey, showing the blind alleys as well as the final route.

## Forecasting mortality: Development of the Lee–Carter model

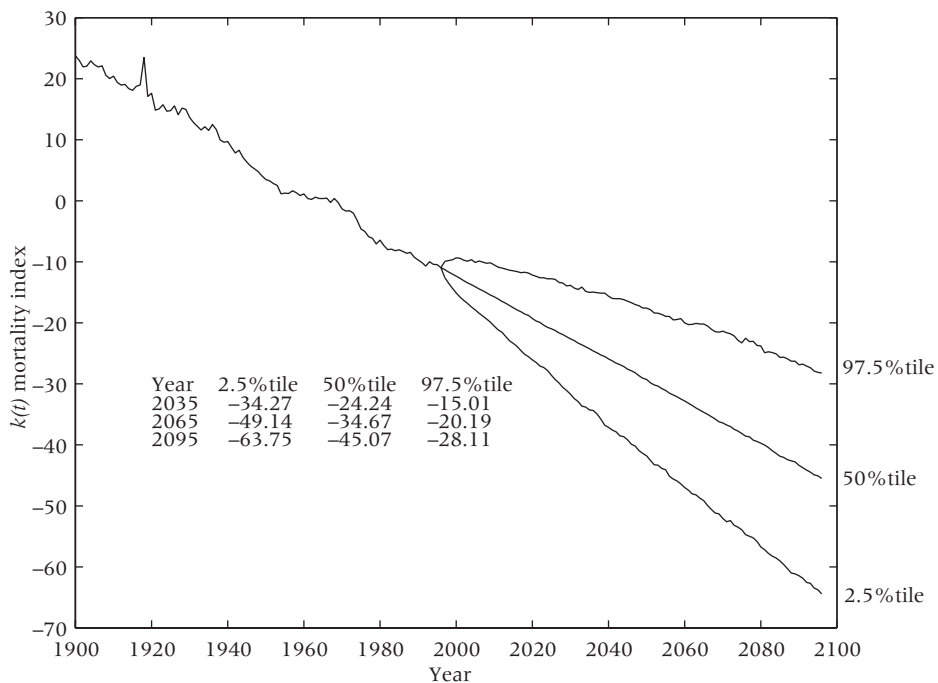
Since US life expectancy was widely believed to have reached its natural upper limit in the mid-1960s while I was a graduate student in demography, I didn't pay much attention to it at the time. Subsequently, mortality began to decline rapidly, and mortality caused larger errors in Census Bureau projections in the 1970s than fertility, so the picture changed. In the late 1980s, Lawrence Carter spent a semester visiting at Berkeley. We had collaborated

**FIGURE 1 The development of demographic and fiscal stochastic forecasting: A schematic view**



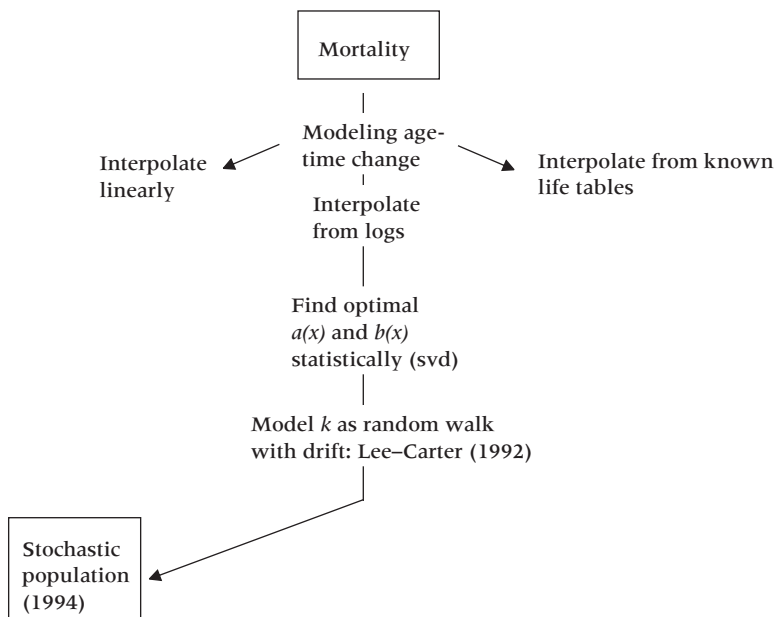
earlier on a model of population renewal based on birth-to-marriage and marriage-to-birth transitions. This time, we decided to use the simple age-time model I had developed for historical work to model and forecast mortality. The single parameter  $k(t)$  indexed the intensity of mortality, and we estimated it for the United States from 1900 to 1989, as shown in Figure 2. Remarkably, the trajectory of  $k(t)$  was quite linear, unlike life expectancy, which rose at a slowing pace throughout the century. This linearity was striking, because it persisted through important changes like the development of antibiotics and the emergence of the AIDS epidemic. We modeled  $k(t)$  as a random walk with drift, and forecasted it along with its probability distribution. From this, we derived the forecasts and probability distributions of age-specific death rates and life expectancy. While we were developing this method, McNown and Rogers (1989, 1992) were developing an approach based on fitting the Heligman–Pollard model mentioned above to the historical mortality data, and then using time series methods to forecast a subset of the estimated parameters. There were lively debates about the relative merits of the two methods. Figure 3 charts this and subsequent efforts to model and forecast mortality. The path ultimately followed is shown by the continuing main line, while the spurs branching off to the sides represent paths tried but abandoned.

**FIGURE 2** Lee-Carter mortality index  $k(t)$ , fitted (1990-96) and forecasted (1997-2096)



SOURCE: Lee and Tuljapurkar (2000).

**FIGURE 3** False starts and promising paths for modeling mortality change



Even before our article (Lee and Carter 1992) was published, Statistics Canada invited us to apply our method to Canadian data, at Nathan Keyfitz's suggestion (Lee and Nault 1993), and they began to use it partially in their projections. Since then, the model has gradually gained acceptance, and is now also used in some respects by the US Census Bureau, Japan, and the United Nations. Tuljapurkar, Li, and Boe (2000) applied it to the G7 countries and found that, as for the United States, it predicted life expectancy gains by 2050 that were two to four years greater than the official projections, and for Japan, eight or nine years greater. Lee and Miller (2001) investigated hypothetically how the method would have worked had it been applied in earlier years. We found that the forecasting errors would have been quite well described by the probability distributions generated by the method. We also found that longer term forecasts tended to understate the future gains in life expectancy, in the United States and in a number of other countries. Recently, Li, Lee, and Tuljapurkar (2004) have extended the method for use in countries with mortality data available for only a few irregularly spaced periods.

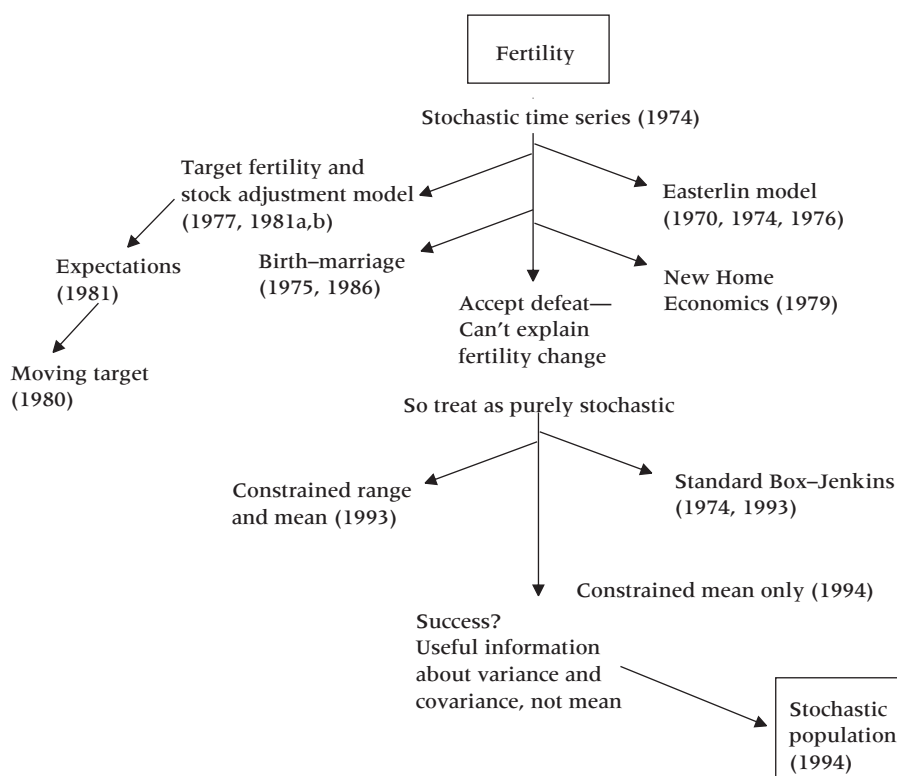
## Forecasting fertility

For fertility forecasts, I reasoned that once the fertility transition was over, we really had no clue about which way fertility would move, and the best we could do was to model its level as a trendless (covariance stationary) stochastic process (Lee 1974a). I then modeled births as an autoregressive time series, with net maternity rates as the autoregressive parameters, and derived the variance of the best linear forecast of the number of births in relation to the uncertainty of the fertility process by analyzing the renewal equation. From this I saw that the uncertainty in the forecast of births grew linearly with time to a good approximation, and that it was a moving average of past fertility shocks with weights equal to the progeny of a birth cohort at each lag. This work was my first presentation at the Michigan brown bag seminar series as a starting assistant professor there, and it was not well received, to say the least. The sociologists thought it was blasphemous to model fertility as a random process. Twenty years later, I got a very similar reaction when I submitted a related paper to *Demography*, which was rejected. The work appeared elsewhere (Lee 1993) and is a key component of the full-scale stochastic population forecasts.

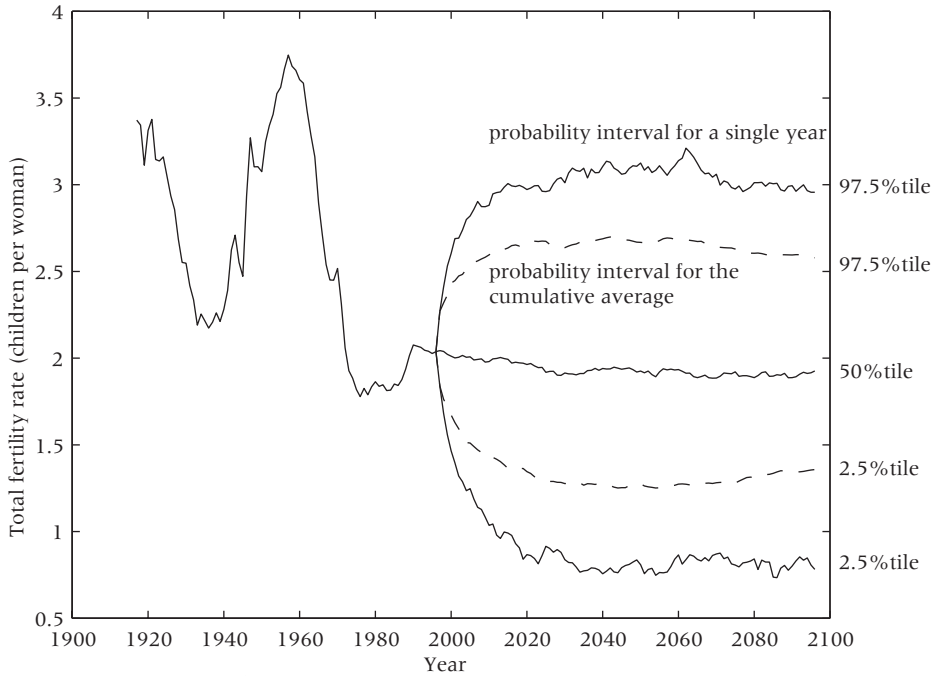
In between my first paper and this elaborated model twenty years later, I had tried many other approaches to forecasting fertility. One line of work modeled an Easterlin-style effect of population age distribution on fertility (Lee 1974b and 1976). In this approach, I used the same kind of autoregressive birth equation, but now fertility was a function of the numbers of earlier births at each lag—that is, it depended on the contemporary population age distribution. Unfortunately, the future did not oblige by conforming to the

predictions of the model; no new baby boom occurred, although a brief upswing between 1988 and 1992 looked promising. I also wrote a series of papers on the use of birth expectations data from surveys (Lee 1981), which led to a paper on “Aiming at a moving target” (Lee 1980), but in the end proved of little use for forecasting. In other work, I considered using New Home Economics type fertility models for forecasting, but did not see how that approach could lead to useful predictions, since it seemed to imply fertility decline without limit as income and female wages rose. Finally, with Carter, I developed a time series model of the joint evolution of births and marriages, each feeding into the other (Carter and Lee 1986). It was an elegant paper, but with the rise in extramarital fertility, marriage became less important as a fundamental force driving fertility, and that approach was abandoned. It was a defeat to come back to treating fertility as a stochastic process, with model forecasts heavily conditioned by imposed assumptions for central tendency and, perhaps, for upper and lower bounds (e.g., total fertility rate between 0 and 4). But sometimes it is best to acknowledge defeat, and make peace on the best available terms; that was what I did in Lee (1993). This long journey of discovery for fertility modeling is portrayed in Figure 4.

**FIGURE 4 False starts and promising paths for modeling fertility change**



**FIGURE 5 Total fertility rate, historical values (1917–96) and forecasted (1997–2096), with 95 percent probability intervals for annual values and for the cumulative average up to each horizon**



SOURCE: Lee and Tuljapurkar (2000).

Figure 5 shows a fertility forecast for the United States treating fertility as a stochastic process. The probability fan seems too wide, with the 95 percent probability bounds ranging from a TFR of .8 to 3.0. However, those bounds are intended to cover annual ups and downs. If we instead ask for central tendencies, by computing the probability interval for the average TFR along stochastic trajectories up to each horizon, we get a narrower 95 percent bound ranging from 1.4 to 2.6 for long-run forecasts.

### The traditional treatment of uncertainty in population forecasts

The traditional approach to estimating and communicating the uncertainty of population forecasts is through the construction of scenarios. First, the analyst constructs high, medium, and low projections for each of the rates, typically fertility, mortality, and net immigration. The high and low trajectories do not have any probabilistic interpretation, but are chosen to span a range that the analyst believes to be plausible. The next step is to bundle these trajectories together to form high, medium, and low scenarios. This is done in

different ways, depending on the purpose of the projections. For example, in the United States, the Census Bureau bundles together high fertility and low mortality (and high immigration) to form the high scenario, yielding the highest rate of population growth, and similarly for the low scenario. But the Social Security Administration bundles together high fertility with high mortality (and immigration), because these generate the lowest old-age dependency ratio (OADR), which is the key demographic variable for their financial projections. The Census Bureau's choice of bundles minimizes the high–low range of the OADR, because high fertility tends to make the population young, while low mortality tends to make the population old, and these effects cancel. The Social Security Administration's bundles minimize variations in the population growth rate, because high fertility makes the growth rate high, while high mortality makes it low, and these effects cancel.

This problem is illustrated by the 1992 Social Security population projections (Board of Trustees 1992). For 2070, the high–low range for the population aged 0–19 years is  $\pm 34$  percent; for the population 20–64, the range is  $\pm 20$  percent; and for 65+ it is  $\pm 9$  percent. Yet for the total dependency ratio, which is the sum of the first and the third divided by the second, the range is only  $\pm 5$  percent, whereas we would expect it to be many times this large. The scenario method inevitably gives probabilistically inconsistent indications of uncertainty for different population variables in the same forecast.

The arbitrariness of the choice of bundles is one of the problems with the scenario approach. Another is that no probabilities can be attached to the high–low range of the projections. Still another is that along the scenario trajectories, fertility is always high, medium, or low, and mortality likewise. It is therefore implicitly assumed that forecasting errors for fertility and mortality are perfectly correlated over time, so that fluctuations such as the baby boom are ruled out by assumption. In addition, errors in fertility and mortality are implicitly assumed to be perfectly correlated with one another, in the sense that high fertility is always associated with low mortality (or always with high mortality). A final problem is that the high–low bounds cannot have any consistent probabilistic interpretation across different measures, since true uncertainties tend to cancel in larger aggregates like total population, relative to their constituent parts like numbers of births in certain years or the sizes of particular age groups. In the scenario approach, no such cancellation can take place.

From this point, I will focus on the main direction taken in the research, with less attention to the side routes that turned out to be deadends.

### Stochastic population forecasts

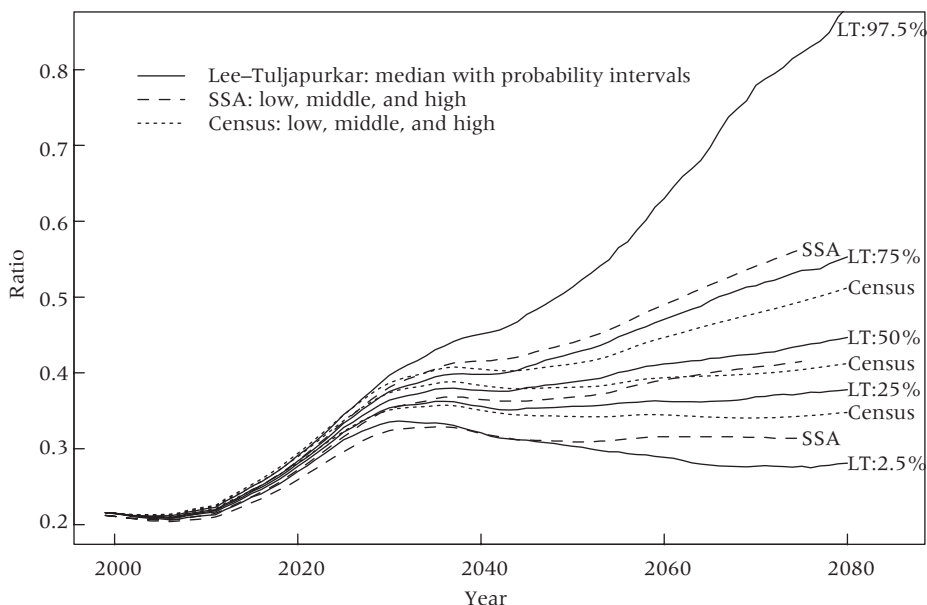
As a graduate student, I wanted to develop more genuinely probabilistic population forecasts. I realized that most population forecasting errors derived

from errors in forecasting the vital rates rather than from individual-level uncertainty, as discussed earlier. At first I thought that the answer lay in formulating stochastic models of fertility and mortality, and then using the probability intervals from their fertility and mortality forecasts to set the upper and lower 95 percent probability bounds for the projection scenarios. However, it soon became clear that this would not do, for all the reasons given above: scenarios do not and cannot work probabilistically. Probabilistic projections required a population projection matrix with stochastic rates that could vary after every projection step, with the probability distribution of the projected population derived from the sequence of matrixes.

I could write down the equations for the stochastic vital rates, perhaps, but I did not know how to derive their implications. A few years later, I began seeing a series of papers by Shripad Tuljapurkar (Tulja), on population renewal in random environments. He was interested in the population dynamics of all species, and he explicitly developed the probability distributions of population variables when reproduction and survival were disturbed by climate, predation, and other partially natural influences (Tuljapurkar 1990). Later, Tulja spent some time at Berkeley and we joined forces to tackle the problem of stochastic population forecasts, combining my work on modeling the vital rates as stochastic processes and his work on population renewal in random environments.

After several years of work, and improvements due to referees, the results were published in Lee and Tuljapurkar (1994). The paper contained analytic approximations for the actual probability distributions, derived at great cost. In order to check on these analytic results, Tulja also carried out stochastic simulations, which confirmed their accuracy. This exercise taught us that while the explicit mathematical solutions were intellectually satisfying and yielded some insights, the stochastic simulations were far simpler and could be used to estimate probability distributions for any desired functions of the age distributions. We reluctantly abandoned the analytic solutions in our subsequent work. This work, finally brought to a successful resolution, drew on earlier deep theoretical research by Tulja in mathematical population biology, and by me in historical demography.

Figure 6 shows forecasts from 1999 to 2080 of the old-age dependency ratio, here defined as (population 65+)/ (population 20–64), with 95 percent probability intervals and comparisons to the Census and Social Security (SSA) projections and ranges. The central forecasts of all three are quite similar, although ours are slightly higher because we forecast more rapid mortality decline. Our 95 percent interval is much broader than the high–low interval of Social Security, which is in turn substantially broader than that of Census, because of the bundling choices for defining their trajectories, as discussed earlier. We find that the high Social Security trajectory is just above the 75th probability bound, and far below the 97.5 percent probability bound. According to our forecasts, there is a considerable possibility

**FIGURE 6 Old-age dependency ratio forecasts: 1999 to 2080**

SOURCE: Lee and Edwards (2001).

that population aging may be much more severe than that considered by Social Security and Census.

I have emphasized the contributions of Tuljapurkar and myself, but at the same time Juha Alho and his collaborators (Alho and Spencer 1985; Alho 1990, 1997) were working along similar lines and making important contributions. Keyfitz, in unpublished work, suggested a different approach: randomly sampling rates of mortality change from the past, and independently randomly sampling levels of fertility, as a basis for developing a stochastic forecast. Stoto (1983) and Keyfitz (1981) developed methods for attaching probability intervals to forecasts of population growth rates and population size by analyzing ex post forecasting errors, an approach extensively developed and discussed in National Research Council (2000). Pflaumer (1988) made stochastic population forecasts by randomly sampling vital rates from the high-low range in official forecasts. Lutz, Sanderson, and Scherbov (1996) have elaborated on this approach in a series of papers, an approach criticized by Lee (1999) and Tuljapurkar, Lee, and Li (in press). Vigorous debates at several international meetings have explored the relative merits of these approaches.

### But what are these good for? Stochastic Social Security forecasts

We had achieved an important goal, one I had started working on 27 years earlier. But we soon realized that nobody really understood these new pro-

jections, or what they were good for. Yes, we could now provide a probability distribution for the forecast of any demographic quantity, simply by examining our stochastic simulation results. But how did, say, the 95 percent probability bounds differ from the standard high, medium, and low scenarios? In fact they differed profoundly, but this was difficult to convey.

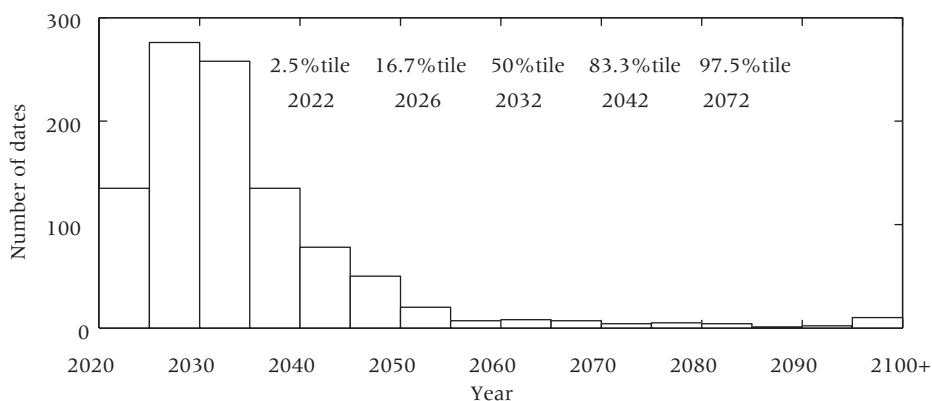
After trying to explain all this to practitioners who were accustomed to the traditional scenarios, and failing to get the point across, we decided that we would need to work out an application ourselves, to illustrate the power of the new stochastic population forecasts. Social Security finances seemed the best place to start, because the Trustees projected over a 75-year horizon every year, and because demography and population aging played key roles in these projections. Furthermore, the Social Security Trust Fund was the cumulation of net surpluses, and so it should depend on the sum of functions of the demography. Along any stochastic trajectory, there should tend to be some degree of cancellation of errors in our projections, but not in the traditional high-low scenarios used by the Social Security Actuaries.

Tulja and I initially developed simulations of the Trust Fund, with only the demography stochastic. Michael Anderson, who at the time was a graduate student in demography and statistics at Berkeley, programmed the stochastic simulations. Soon, however, we moved on to model productivity growth rates and real interest rates as stochastic processes, and for some purposes we similarly modeled stock market returns. Our stochastic forecasts for the Trust Fund typically reflected four stochastic inputs out of the eight or ten inputs that were usually viewed as uncertain. The inputs we did *not* treat as stochastic included inflation, disability, and immigration. We believed that the four we included were the most important sources of uncertainty and would capture most of the overall uncertainty.

Figure 7 shows histograms for the date of exhaustion of the Social Security Trust Fund assuming no change in the payroll tax rate, no investment of the fund in equities, and no change in the currently legislated increase in the normal age of retirement to 67, based on the central assumptions contained in the Trustees' Report of 1998 (somewhat more pessimistic than the more recent Trustees' assumptions). The median date of exhaustion was then 2032, with 2.5 percent probability of exhaustion by 2022 and 97.5 percent by 2072. We consistently find that even the most favorable 2.5 percent bound shows exhaustion in less than 75 years, in contrast to the Trustees' "Low Cost" forecasts, which suggested that if we were lucky the system would be able to continue robustly in the future.

At the same time we were developing our stochastic forecasts for the Social Security Administration, the Congressional Budget Office (CBO) had noted our 1994 paper on stochastic population forecasts and asked whether they could use our stochastic population trajectories as the basis for stochastic Social Security projections of their own. We sent them a set of a thousand stochastic population simulations, and they did indeed develop

**FIGURE 7 Histograms of 1,000 dates of exhaustion for the Social Security Trust Fund**



SOURCE: Lee and Tuljapurkar (2000).

stochastic forecasts that they published annually for a number of years. In 2001 they published a more elaborate stochastic forecast for SSA, this time developing their own stochastic population model from scratch. I believe that this effort benefited from the short class we taught in Washington, DC for three summers, which I will describe later.

In 1999–2000, I served on the Technical Advisory Panel for Social Security, and I presented our results there. The Trustees and Actuaries had already been advised by earlier committees that they should do stochastic forecasting, so our efforts fit well with those recommendations. Although nothing happened at the time, in 2003 SSA did develop and publish its own stochastic forecasts for the first time, using methods closely related to ours. They also published comparisons of their stochastic projections to ours and to a set developed by CBO. The three were remarkably consistent.

### Stochastic fiscal projections

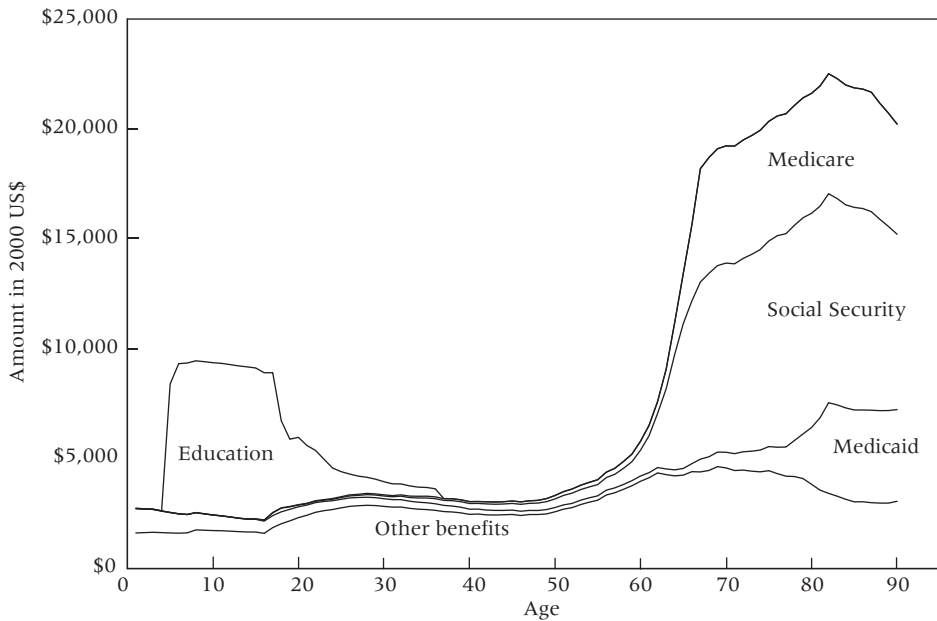
When President Bush began arguing for tax cuts in early 2001, the Congressional Budget Office was projecting large surpluses over the next decade, and Alan Greenspan, Chairman of the Federal Reserve, was worrying about what would be done once all the government debt had been paid back. A probabilistic forecast would have shown that not much confidence should have been placed in those projections, and indeed the Congressional Budget Office had included probability intervals based on its own analysis of the past performance of its projections, and these showed that it was quite possible that the surpluses would turn to deficits within a few years. I was invited to testify to the Senate Budget Committee about the uncertainties in the fiscal outlook

and on the impact of population aging on the federal budget. My testimony was received with interest, but I doubt that it had much impact. Here is the story of how the probabilistic fiscal forecasts were developed.

In 1995, I served on a panel organized by the Committee on Population of the National Academy of Sciences, chaired by James Smith, to assess the economic and demographic consequences of immigration to the United States for a special bipartisan Congressional Commission. My task was to estimate the fiscal impact of immigrants using a longitudinal design which required that I prepare long-run projections of US government budgets at the federal, state, and local levels. I had developed relevant methods during earlier work on estimating the externalities to childbearing, involving the estimation of age profiles of government benefits and taxation and shifting these over time with productivity growth. Figure 8 illustrates cross-sectional age profiles of this sort for the year 2000, showing the dominating importance of public education, Social Security benefits, Medicare, and institutional Medicaid. Similar age profiles were estimated for various kinds of taxes.

I received advice on modeling government budgets from Alan Auerbach and Robert Inman, two leading public finance economists also on the immigration panel. I hired Ryan Edwards, then a graduate student in Economics, to work on the budgetary side of these projections. Tim Miller, a demographic

**FIGURE 8** How the value of benefits received from government programs varies by age of individual in the US in 2000 (lines show cumulative totals)

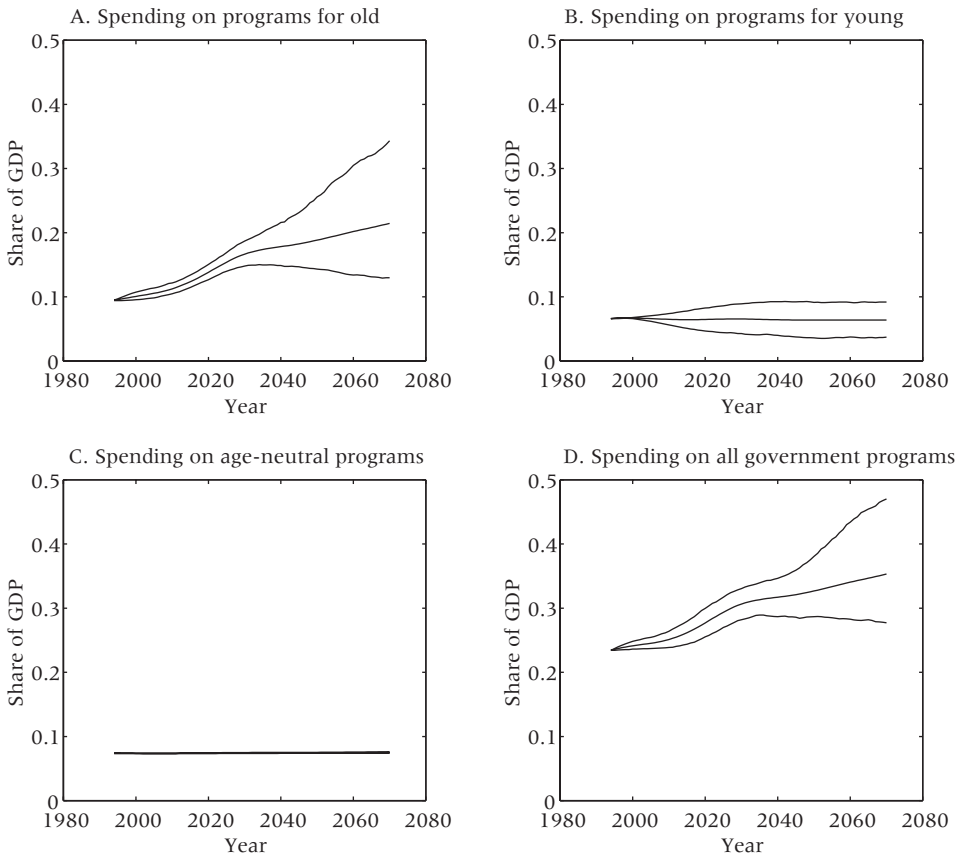


SOURCE: Lee and Edwards (2001).

researcher at Berkeley, did most of the necessary estimation. With NIA support, this research team worked for a year and brought the project to an interesting and successful conclusion. As a byproduct, we had developed the expertise to make long-run deterministic budget projections for all levels of government. If we could do it for one set of demographic and economic assumptions, then we could do it for others, so we had the machinery in place to construct stochastic budget forecasts for all government programs and total taxes and expenditures, exploiting detailed population age distributions and schedules of benefits and costs by age, as driven by productivity growth rates. We were able to draw on both the stochastic population projection methods and the methods for stochastic projections of SSA. It seemed only natural to take advantage of the situation to construct probabilistic forecasts of federal, state, and local taxes and expenditures. We did this, based on seven kinds of taxes and 25 age-specific government programs. The results were published in Lee and Edwards (2001 and 2002).

Figure 9 shows probabilistic forecasts of government spending as a share of gross domestic product at all levels (federal, state, and local) combined, but disaggregated by age group in the first three panels and in total in the last panel. These forecasts are conditional on the assumption that current program structures remain constant or vary only according to currently legislated plans such as the increase in the normal retirement age. Panel C shows that no change is projected for the share of age-neutral programs, consisting mainly of such items as defense expenditures, police, fire, and research: expenditures that cannot be assigned to recipients of any particular age. The flat line with no probability dispersion reflects the assumption made, following assumptions of the Congressional Budget Office, that such expenditures will be a constant share of GDP in the future. Panel B shows that expenditures on the young (defined on a programmatic basis and including expenditures on higher education) are also expected to be flat over the coming decades, although in this case there is substantial uncertainty, reflecting uncertainty about fertility and therefore the share of children in the population. Panel A shows that expenditures on the elderly (again defined by program) are expected to rise strongly, nearly tripling in 80 years, owing primarily to the effect of population aging on Social Security, Medicare, and institutional Medicaid (that is, for long-term care). The probability distribution is narrow for the first 25 years or so, reflecting mainly uncertainty about mortality and survival. After this point, however, the much greater uncertainty about fertility begins to affect the projected size of the working age population, which drives projections of GDP and therefore strongly influences expenditures on the elderly as a share of GDP. Panel D shows the forecast for expenditures for all age groups combined. We would expect a negative correlation between expenditures on children and the elderly as a share of GDP, since variations in fertility would affect these

**FIGURE 9** Projected government spending as a share of GDP by broad age group of recipient, 1994–2070 (mean and 95% probability intervals)



SOURCE: Lee and Tuljapurkar (2000).

expenditures in opposite directions. This correlation is implicitly taken into account in the probability intervals in Panel D. The central forecast shows that total governmental expenditures as a share of GDP would rise by more than 50 percent over the next 75 years, which we have seen is due entirely to expenditures on the elderly. The 95 percent probability interval extends from about 28 percent of GDP to about 48 percent of GDP, with uncertainty in the upward direction being greater than downward.

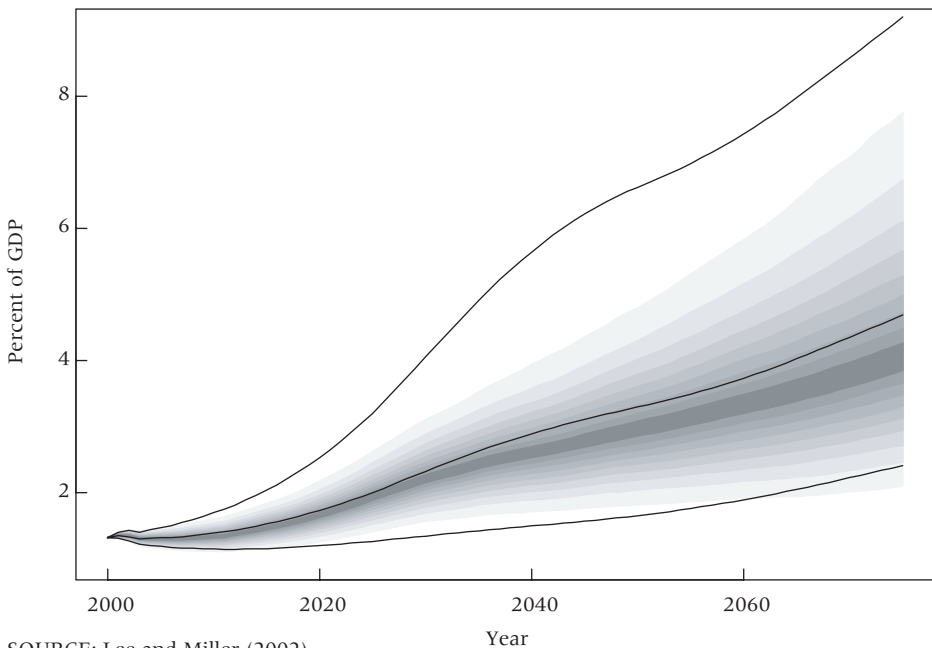
### Health care costs: The joker in the deck

Medicare expenditures were one piece of the federal projections reported above about which we had reservations. However, based on earlier research by actuaries of the Health Care Financing Administration, Tim Miller (2001)

published a paper showing how data on Medicare costs by time until death could be used in projections of Medicare costs. In these projections, as mortality falls, there are two effects on costs. First, there are more old people at every age, so costs tend to rise. Second, at any given age a smaller proportion of people is near death, so costs tend to fall. These two effects largely cancel, as it happens, so it makes little difference to costs whether mortality declines rapidly or slowly. Miller's work paved the way for a subsequent paper on stochastic forecasts of Medicare expenditures (Lee and Miller 2002). In these forecasts, we estimated a stochastic time series model for the growth in Medicare expenditures per enrollee in excess of per capita income growth. This was used as a multiplicative shifter for a schedule relating Medicare costs to years until death. Along each stochastic trajectory, we knew the distribution of deaths by age, so we could apply this schedule to the distribution of deaths to find the Medicare costs implied.

Figure 10 contrasts our probabilistic time-until-death-based forecasts of Medicare costs with the official government projections. Here, probability deciles are indicated by the darkness of the fan. The probability is 10 percent that Medicare hospital insurance spending will fall in the darkest area and 90 percent that it will fall within the whole shaded area. The lines represent the

**FIGURE 10 Medicare Hospital Insurance Program as a percent of GDP: Lee–Miller probability deciles and Trustees' scenarios (excludes Medicare's Supplementary Medical Insurance Program)**



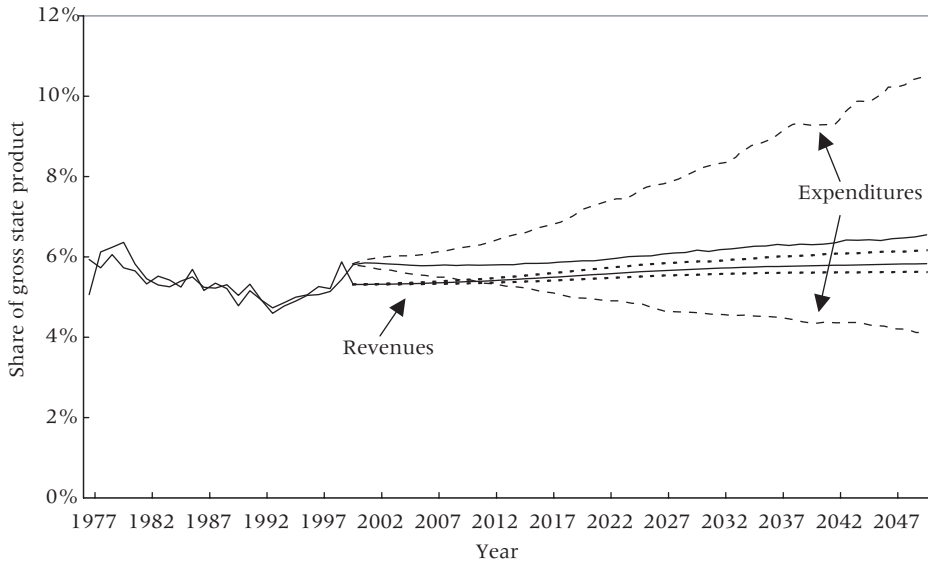
SOURCE: Lee and Miller (2002).

Trustees' low, intermediate, and high cost scenarios. It turns out that probabilistic projections are not always more gloomy than deterministic ones. First, taking time until death into account leads to projections of Medicare costs that are substantially lower than the official projections by the Medicare Trustees, because our forecasts implicitly assume improving health at every age as mortality falls. Second, we see that while our lower probability decile corresponds closely to the Trustees' low projection, their high projection is far more pessimistic than our upper 97.5 percent bound: we find it very unlikely that their high scenario will come to pass.

### Population and fiscal projections for California

In 2001, I was asked to prepare projections of population aging in California for the state legislature, and I took advantage of the opportunity to enlist Tim Miller and Ryan Edwards in the effort, and to prepare stochastic projections for the population of California and for its budget through 2050. Miller developed stochastic immigration and internal migration projections, which we incorporated in the demographic forecasts. Figure 11 shows our projections of the state budget, again assuming current program structure. In contrast to the federal and general government projections, we find almost no systematic tendency for expenditures to rise relative to gross state product, because for the most part the state does not provide benefits tar-

**FIGURE 11 Projections of California's general fund revenues and expenditures as shares of gross state product**



SOURCE: Lee, Miller, and Edwards (2003).

geted to the elderly and therefore is not affected by the projected population aging. Revenues are almost flat with almost no uncertainty, since they are expressed as a share of gross state product and are not much affected by population age distribution. These projections attracted considerable interest in Sacramento, and point toward the application of these methods at the state and local level.

### Proselytizing and training

In the summers of 1998 to 2001, Tulja, Edwards, Anderson, and I taught a short intensive class in Washington, DC on our new methods for stochastic forecasting, with funding from the Social Security Administration. We pitched it to professional government forecasters, and people attended from many government agencies, including Social Security, Census, Congressional Budget Office, Office of Management and Budget, General Accounting Office, and Veterans Affairs. Some graduate students, academics, and government forecasters from other countries attended as well. The lectures seemed to generate a great deal of interest. It is difficult for us to assess the impact of these classes, but in 2001 CBO published its own stochastic forecast for Social Security, and in 2003 SSA published its own version, as discussed earlier.

### Where next for stochastic forecasting?

These general methods for making stochastic forecasts of mortality, fertility, population, Pay-As-You-Go pension systems, health care costs, and full government budgets appear to be reasonably well established, although there is plenty of room for improving each, and for trying completely different strategies. Many questions remain about the best way to handle various details, and work on these questions is ongoing. Doubtless problems will be discovered and the methods will evolve. For example, a book by Tabeau, Jeths, and Heathcoate (2001) discusses many approaches to forecasting mortality including Lee–Carter, and an unpublished manuscript by Girosi and King (2004) contains a searching critical analysis of the Lee–Carter method and develops an alternative approach based on covariates and smoothness priors. Denton and Spencer in Canada, and CBO and the Social Security Actuaries are all exploring alternative approaches. Li Nan and I are working on modifying and extending Lee–Carter for use by the United Nations in the mortality component of their global population projections.

I would like to see government agencies develop and use stochastic long-run budget projections. This seems particularly important, given the great stresses that population aging will put on the budgets of the industrial world through public pension programs, health care, and long-term care. Policy changes today should be undertaken in light of this sobering long-

term outlook, but also with a full appreciation of the degree of uncertainty about these pressures. In recent years, both the Bush administration tax cuts and the Iraq war illustrate the importance for good decisionmaking of taking into account not only the best guess forecast, but also a careful assessment of the degree of uncertainty about that guess, and the expected cost of errors.

One of the most challenging questions in this area is just how policy-makers should take uncertainty into account: by acting quickly to build up buffer funds or to contain a situation that might deteriorate if no action is taken, or by postponing action until we have a clearer idea of which direction the cat is going to jump. A useful start has been made on this question by Auerbach and Hassett (2000). But posing the problem as I just did suggests that once the cat jumps, we will know how the future will unfold—rapid versus slow gains in life expectancy, deficits versus surpluses, and so on. In reality it is much more likely that the future will be no more certain in ten years or twenty or fifty than it is now. The world is not going to choose a direction and then adhere to it thereafter. There is uncertainty about big changes and at every step of the way, without end. We must live and act in the face of this uncertainty as we have always done, but understanding it better should lead to better decisions.

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