

Order, Divine and Profane
A Reflexion on Petty, Graunt, Süßmilch, Euler,
and
Demography in 2012

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When I was being taught German, I had to pronounce passages over and over. One of these passages began,

“Um zu begreifen, wie es damals war ...”

“In order to grasp how it all was in those days...”. I do not recall what philosopher of history wrote the passage – perhaps you can tell me – but it appeals to me. Nicht “was damals war” sondern “wie es damals war”. Not, “what was going on”; rather “how it all was”. The air, the feel of it, the sense of things, the scale. Ideas as instincts. That is what I want from historical studies.

I shall not be presenting original historical research today. Instead, I draw on what I have learned from studies by others and especially from Hervé LeBras’ wonderful book *La Naissance de la Mortalité*. It is a gripping detective story, as it taps Sir William Petty rather than John Graunt as the author of Graunt’s *Natural and Political Observations ... Made Upon the Bills of Mortality* whose February 1662 appearance we celebrate today. The title of the volume mentions both “Natural” and “Political” observations. I am leaving to Hervé the political,

and I shall concentrate on “natural”. I shall regard the volume as a work within a tradition of “natural philosophy”.

Hervé’s analysis of the *Observations* puts us back into the mindset of the author, the author’s circle, and the author’s time. He lets us see the first model lifetable, the calculations that initiated mathematical demography, being crafted with the tools and habits of the century.

What century? For our purposes today, I should like to let the century run from 1662 through 1761, so as to take in the other great beginning for mathematical demography, the launching of stable population theory in the partnering of Johann Peter Süssmilch and Leonhard Euler. My title for this reflexion harks back to the title chosen by Süssmilch, *Die Göttliche Ordnung*, the Divine Order. The second, 1761 edition presents calculations supplied by Euler, a population renewal process converging to exponential growth. It reflects Euler’s discovery in 1760 of the stable age pyramid and what we now call the Euler-Lotka Equation for intrinsic population growth. My title also mentions Demography in 2012, the proper 350th anniversary of 1662. We are only in 2009, but on our timescale near enough to 2012 as makes no difference.

The full title of Süssmilch’s book reads *Die Göttliche Ordnung in den Veränderungen des menschlichen Geschlechts, aus der Geburt, dem Tode, und der Fortpflanzung desselben erwiesen*. (The Divine Order Manifested in the Changes of the Human Race from Birth, Death, and Procreation). These words summon up a web of beliefs and associations which have now become remote to most of us, which need parsing for our students, but in which Petty, Graunt, Süssmilch, and Euler were all inevitably

immersed. My subject today is something that was easier for them that it has become for us. The scientific quest for order is shaped by beliefs about the origins of order. For our century, beliefs about the origins of order have given way to puzzlement about the origins of order.

I shall be talking here about order in a small way, about demographic regularities in lifetables and population trajectories, about macro-demographic modeling and its troubled relationship with micro-demographic modeling. But I have in my mind larger questions about the origins of order, as we share them with physicists, biologists, philosophers, and all folks. My aim is to take up strands from our historical understanding of 1662 and 1761 and ask how the world we have lost illumines the world we have gained.

The title *The World We Have Lost* (1965) brings us to Peter Laslett, whose spirit presides over our celebration here. Peter edited the facsimile edition, *The Earliest Classics: Graunt and King*. Peter brought together demography, philosophy, and history as we are seeking to do. His writings set the standard for empathetic recovery of the conditions, presuppositions, and sense of the world of earlier times, including the face-to-face pre-industrial society in which Graunt and Petty lived. I reach back to him for moral support when I encounter historians bent on relentless anachronism, importing the values and assumptions of the present into the past in order to condemn and disown the past.

T.S. Eliot referred in *The Dry Salvages* to “...superficial notions of evolution, which become[] in the popular mind a means of disowning the past...” For us, on the contrary, evolution and

evolutionary demography will be one way of owning the past.

Let us turn now to 1662. The lifetable in Graunt's *Observations*, the first explicit lifetable, is a model lifetable. It is a model lifetable in the full sense of the tables that dominated demography in the later Twentieth Century. Two benchmarks, cohort survivorship to ages 6 and 76, are drawn ostensibly from observation and serve as parameters. Survivorships and cohort deaths in intervening ten year intervals of age are filled in from a mathematical formula. The author says "we sought six mean proportional numbers ...". Simplistically, we might interpret the formula as providing for constant hazards and exponentially declining survivorships between 6 and 76, but Hervé LeBras has shown that the algorithm is subtler and grounded in Seventeenth Century practice for multiplicative calculations.

The essential feature is that cohort size declining with age is taken to be a *multiplicative* process. Constant hazards instead of curving hazards is a secondary assumption, later easily relaxed. It is the recognition of the multiplicative character of population change that is portentous. This recognition hovers between the *a posteriori* and the *a priori*. In a sense it is an observable regularity, as ratios of deaths to those at risk of dying, ratios of flows to stocks, vary less than counts from to age, case to case, and time to time. In another sense, it is a postulate vindicated by the rich mathematical structures that proliferate from it, appealing like a symmetry or a principle of design.

Arithmetic series were mastered by the ancients, as we learn from Hervé LeBras (2000: 217–230), but geometric series, multiplicative processes, and exponential curves, were exciting, open to discoveries, in 1662. I remember my own amazement as a

schoolchild at the power of exponential growth illustrated in an assembly talk. This amazement speaks to us in the table of predicted populations Süssmilch published from a letter of Euler's. As exponential growth sets in, with a posited Net Reproduction Ratio of 3, population counts rise from a single primordial couple to 4 million in 300 years. Süssmilch notes that after 450 years, one could have 3 billion people, "Mehr als jetzt wirklich auf dem ganzen Erdboden leben...": "More than now truly live on the whole globe."

The mathematics leads Süssmilch (1761:299) to observe that, after Noah's flood, population growth must have been restricted through epidemics and other disturbances, shorter lifespans and lower fertility. Otherwise the increase would have been "faster than the condition of the already populated world could have sustained."

A cohort is a population, a closed population depleted by deaths but not augmented with further births after the births of the cohort members. The multiplicative process of cohort survival, expressed in the 1662 model lifetable, is a special case of the multiplicative process of growth in population size. The Graunt table arranged by age and the Euler table arranged by time are formally parallel.

As I reflect today about demographic order, I shall concentrate on model lifetables. But with more time I would sketch a parallel account of mathematical models of population growth. I would consider how demographers have tried to account for long-term trends, for instance across prehistory, and for large scale swings, for instance in Twentieth Century baby booms. We want to explain how the individual behaviors studied in mi-

crodemography culminate in large-scale aggregate phenomena studied in macrodemography. Current research into origins for this kind of demographic order point to processes of feedback which channel randomness into structure. But our understanding is incomplete.

I turn again back to lifetables. When I teach the Graunt model lifetable, I begin by asking why the table closes with zero survivors at 80. The answer I listen for (and do not always hear) is not something about Petty's or Graunt's observations of old people, but the verse from Psalm XC, "The days of our years are threescore years and ten; and if by reason of strength they be forescore years, yet is their strength labor and sorrow; for it is soon cut off, and we fly away." I then ask about the phrase, "Of one hundred quick conceptions..." and ask the class "What are quick conceptions?" "Where do you hear of 'the quick and the dead'?"

Then I talk about the founding of the Royal Society in 1660, Petty and his friends as founding members, Graunt elected, and I talk about the plague and Isaac Newton being sent for safety home from Cambridge, and having an apple fall on his head. The point I want students to carry away is that model lifetables, under their early name, "Laws of Mortality", are to be seen at their origin like Newton's Law of Gravitation or Laws of Motion. Physics, biology, society, all aspects of the created world, could all equally be designed by the Creator along mathematical principles.

In Johann Peter Süßmilch, Pastor of St. Peter's Church in Berlin-Kölln, the regularities of the lifetable found their most enthusiastic admirer. His twenty-second chapter is enti-

tled, “Von der bewundernswürdigen Ordnung der Sterbenden nach dem Alter”: “On the wonder-worthy ordering of the dying by age.” (p. 285). He was not given to understatement.

Here we find general rules, by which the duration of life of all men on the globe is determined with exactness....

.... A feature of the greatest importance, all these proportional numbers are produced by a huge variety of causes, all separate intrinsically variable....

Thus God it is ... who gives each and every one his ending, arising jointly from each one’s particular condition and from each one’s relation to the whole...

* * *

Hier finden wir allgemeine Regeln, nach welchen die Dauer des Lebens aller Menschen auf dem Erdboden auf das genaueste bestimmt ist.. (s. 287, z. 2–5)

... und welches ein höchst wichtiger Umstand ist, es werden alle diese proportionierte Zahlen durch ein grosse Menge Ursachen bewirkt, welche sämtlich abgesondert und in sich veränderlich sind... (s. 289, z. 8–11)

Gott ist es also,...der...einem jeden das Ziel setzt, ein Ziel, das sowol in dem besondern Zustand eines jeden, als auch in desselben Zusammenhang mit dem Ganzen seinen Grund haben muss ... (s. 289, z. 26–32)

Let us now skip across two centuries or so. We pass over the first empirical lifetable by Edmund Halley, the lull in attention

to lifetables, the revival of interest, the mathematical curves of Gompertz and Makeham, the recourse to data by Farr, Rickman, Greenwood, and followers, and we come to the 1950s, 60s, and 70s, and the Age of Model Lifetables. It was a time when possession of Coale and Demeny's (1966) several pound volume *Regional Model Lifetables and Stable Populations* became the identity card for the profession of demographer.

Although the title invokes both Graunt and Euler, I detect a distancing from that tradition, clearer in Coale and Demeny than in the U.N. Model Tables and New U.N. Model Tables that preceded and followed them. The 1966 tables are descriptive summaries of empirical clusters of outcomes, not "regularities" in the strict sense of patterns expressive of rules. Lifetables are not essentially "law-like". Coale and Demeny paste together three formulas with a bunch of arbitrary change points for each table in behalf of statistical fit. Even these summaries based mainly on European historical data turned out to be poor guides to life chances elsewhere, especially in Africa. In place of publishable model tables, demographers began to rely on relational models like Brass's logits which allow spontaneous construction of separate standards for any time across places or any place over time. As data for far-flung places improved, empirical tables took over the field. www.mortality.org became the source-book. Model tables came to have specialized uses in forecasting and elsewhere rather than wide applicability.

But now, in the cycles of history, model lifetables have come back into vogue. With the epidemiological transition and the abatement of infectious mortality in developed countries, mortality is principally old-age mortality, and old-age mortality has come to appear remarkably law-like. Gompertz' 1825 model

makes adult age-specific mortality rates or hazard functions increase exponentially with age. For Petty or Graunt's mean proportional factors, we substitute mean proportional proportional factors, and we come up with curves that fit most adult ages quite well from Rostock to Tokyo or China to Peru.

What is more, over a range of ages, Gompertzian exponentials fit quite well for lions and fruit flies, worms, and sheep. The regularity prevails across species. Where the pattern alters, at extreme ages, it turns out to alter in similar ways across species. As discovered by Jim Vaupel, Jim Carey, and a new generation of biodemographers, many here today, age-specific mortality curves in species with very different body plans and lifestyles taper at extreme ages forming shapes resembling plateaus. Süßmilch would have been delighted at this striking expression of göttliche Ordnung stretching across creation.

Today's biodemographers would not take Süßmilch's stance, exactly. But habits of mind have been inherited, and we count on finding order in demographic phenomena, so much so that we search for it, and we expect order to have an explanation. When we draw back and pause, we feel wonder at the regularities in hazard curves across species. But we are not altogether surprised. Like the world itself, the demographic world is ordered, if not by divine agency, then by profane agency.

For biodemographers, when we ask what has taken the place of divine intention in explanatory frameworks, the theory of evolution comes first to mind.

Let us consider now these commonalities in the shapes of lifetables across species. On the two-hundredth anniversary of Darwin's birth, it seems appropriate to be hunting for the ori-

gin of commonalities in processes of Darwinian natural selection. This impulse to turn to evolutionary demography could be misguided. Alternative attempts to account for Gompertz curves in mechanistic terms have been made. In my view they typically make assumptions which smuggle in the exponential patterns they are intended to explain, but some new one might succeed. Alternative attempts to account for tapering hazard rates entirely in terms of demographic selection have been made. Demographic selection here refers to the culling out of frailer members of a heterogeneous population leaving less frail survivors. In my view, culling does operate but not sufficiently strongly to supply a total explanation. Another alternative is to hold that regularities have been exaggerated and wider knowledge of data will make them fade.

If the regularities in demographic schedules across populations and across species do have a fundamental character, however, then evolutionary models do offer hope of explaining them. We need to explain both diversity and commonality. Elephants live longer than worms, and we want to account for such differences. Elephant hazard functions rise and taper much like those of worms, and we want to account for such similarities. Biodemographers are pressing forward on two fronts. Optimal life-history theory offers to help account for diversity. Mutation accumulation theory offers to help account for similarity.

In life-history theory, the optimization at issue is optimization in terms of adaptive Darwinian fitness, when organisms allocate constrained physiological resources across the life course to processes including growth, maintenance, reproduction, familial or social transfers, and repair. In mutation accumulation theory, senescence is a non-adaptive side effect of genetic load. Each

member of a population carries a load of large numbers of mildly deleterious mutant alleles. Alleles acting at older ages detract less from net reproduction. Natural selection weeds them out more slowly from generation to generation, and more can accumulate before weeding balances the pressure of new mutations. Because the force of natural selection is sensitive to age, highly random, unstructured collections of mutational effects can in principle be converted into highly structured age-specific demographic patterns. That is a hope that propels ongoing research. Evolutionary demography becomes a route to explaining order.

I am, as you see, an optimist about the potential for evolutionary demography to lead us toward explanations of cross-species commonalities, But I am a troubled optimist.

My trouble begins in small problems in day-to-day research, which may go away in a few years. We have been building models in the grand tradition of evolutionary theory where mutations produce very random effects, with no rhyme or reason, hither, thither, and yon. The rhyme and reason come from Darwinian natural selection. The mathematics which ties effects on schedules of fertility and mortality to differences in fitness is an accessible and powerful tool for generating structure out of randomness. It has been our ticket to successful explanations.

But recent progress in genetics seems to be telling us that the genome and the processes that translate genetic code into phenotypes have a much more elaborate architecture than our random models. Gene networks, hot spots, promoter regions, regulatory DNA, epigenetics, everything that is the rage in departments of genetics at the moment, impairs the innocence of the randomness we started by thinking we could assume.

I am troubled because I have been hoping that what we already knew about the mathematics of natural selection could help account for the order we see in demographic schedules across species. But if complex genetic architecture is essential, demographers will have to wait until that architecture is understood and modeled. More poignantly, we push back our search for the origin of order. The origin of order is not to be sought in evolution as we know it, but in some sort of process imprinting order onto evolution.

Such a speculation is part natural science, part philosophy. As demographers, we find ourselves having to be “natural philosophers”, which is what demographers began by being in our founding century. Philosophical puzzlement about the origins of order impinge on us. Questions of the origins of order in the biological world lead quickly back to questions of the origins of order in the physical world. For cosmologists, the origins of order in the physical world remain as much a puzzle as for the rest of us. Order, with its wondrous diversity of forms, is something that so easily might not have been.

It is fashionable in such reflections to intone the words “emergent phenomena”. I do not know what these words mean, so I shall not be able to tell you. The words are meant to convey the idea that structures on large scales can be composed out of processes on small scales without being entailed by them. In our discipline, it sounds like a claim that macrodemography can exist without microdemography, a claim I do not accept. Among the conceptual examples offered for emergent phenomena are simple mathematical algorithms that generate unforeseeably elegant pictures. However, to my mind, while the elegant macroscopic pictures may be unforeseen, for beings smart enough, they

would not be unforeseeable. They would be subject to design. Süßmilch might have been delighted, but are we?

Our search for origins of demographic regularities carries forward the tradition of Petty, Graunt, Süßmilch, and Euler. These are big questions. They are questions about patterns that appear only in the aggregate. They are studied by macrodemographers. But the population of macrodemographers has had a negative intrinsic rate of natural increase (with the consequence of an old age pyramid) while the population of microdemographers expands. Forecasters among us might reasonably predict the impending extinction of the macrodemographic lineage. It seems to me more probable that, like horseback riding or sailing, macrodemography will survive at least as an elite sport. What was once everyone's arduous necessity for going places becomes an excellence to be cultivated.

The founders of Demography whom we celebrate today were immersed in a web of beliefs that fostered their discoveries. Order was an expression of design. Principles of design were newly seen to be mathematical. So designing demographic models on mathematical principles was fitting. Regularities in models mirrored regularities in the creation.

We are remote from the world of the founders. We struggle to recover a feeling for it through historical studies. But the world of the founders also lives on in us. We do their kinds of calculations. We marvel at law-like processes they first highlighted. We carry inside us some of their belief in order.

Some of it, and only some of it. Replacing divine order with profane order, we replace a sense of the origins of order with puzzlement over the origins of order. As demographers we see this predictment in many small ways. A gap persists between

microdemography and macrodemography. Explanations of large scale structure in terms of feedback processes or evolutionary influences almost work, but, it seems, always not yet quite. Our small puzzles put us in mind of large puzzles. Three and a half centuries after the *Natural ... Observations*, looking forward to 2012 and beyond, we are further along at working out the mathematics of the human condition, but not as far as we want to be at knowing what it means.

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